

Theta-Induced Electric Dipole Moment of the Neutron via QCD Sum Rules

Maxim Pospelov and Adam Ritz

*Theoretical Physics Institute, School of Physics and Astronomy
University of Minnesota, 116 Church St., Minneapolis, MN 55455, USA*

Using the QCD sum rule approach, we calculate the electric dipole moment of the neutron induced by a vacuum θ -angle to approximately 40-50% precision, $d_n = 2.4 \times 10^{-16} \bar{\theta} e \cdot \text{cm}$. Combined with the new experimental bound, this translates into the limit $|\bar{\theta}| < 3 \times 10^{-10}$.

The impressive experimental limits on the electric dipole moments (EDMs) of neutrons and heavy atoms in general put a very strong constraint on possible flavor-conserving CP-violation around the electroweak scale [1]. This precision means that EDMs can in principle probe a high energy scale by limiting the coefficients of operators with dimension ≥ 4 , such as $G\tilde{G}$, $GG\tilde{G}$, and $\bar{q}G\sigma\gamma_5 q$ etc.. However, in practice, while these operators can be perturbatively evolved down to a scale of order 1 GeV, the ultimate connection between high energy parameters and low energy EDM observables necessarily involves non-perturbative physics.

The θ -term, $\theta G\tilde{G}$, which has dimension=4 may be interpreted as the lowest dimension CP-violating operator, un-suppressed by the high energy scale. Experimental tests of CP symmetry suggest that θ is small and, among different CP-violating observables, the EDM of the neutron ($d_n(\theta)$) is the most sensitive to its value [3, 4]. However, the calculation of $d_n(\theta)$ is a long standing problem [5–7]. According to Ref. [6], an estimate can be obtained within chiral perturbation theory, relying on the numerical dominance of contributions proportional to $\ln m_\pi$ near the chiral limit. However, there are also non-logarithmic contributions, incalculable within this formalism, which in principle can be numerically more important than the logarithmic piece away from the chiral limit. As a consequence one is unable to estimate the uncertainty of the prediction [6].

There are a number of important incentives for refining the calculation of $d_n(\theta)$. While θ primarily arises as a fundamental vacuum parameter, one may also induce calculable corrections via the integration over heavy fields, and such corrections are significant in theories where the fundamental parameter is absent. One scenario of this type arises as a result of exact P or CP symmetries [8], whose subsequent spontaneous breakdown allows radiative corrections to induce an effective θ -parameter, which is then the dominant source for d_n . Alternatively, the fundamental θ parameter may be removed via the axion mechanism, but then a small but finite effective θ -angle will again survive, induced by the linear shift of the axionic potential due to higher-dimensional CP-odd operators [9]. In this case, the calculation of $d_n(\theta)$ is part of a more generic calculation of d_n .

In this letter, we apply the QCD sum rule method [2] to

obtain an estimate for $d_n(\theta)$ beyond chiral perturbation theory. Within currently available analytic techniques, QCD sum rules appears the most promising approach to this problem as it has, in particular, been used successfully in the calculation of certain baryonic electromagnetic form factors [10, 11]. Within the sum rule formalism, physical properties of the hadronic resonances are expressed via a combination of perturbative and nonperturbative contributions, the latter parametrized in terms of vacuum quark and gluon condensates. We note that previously QCD sum rules were used to estimate the neutron EDM induced by a CP-odd color electric dipole moment of quarks [12, 13]. Surprisingly, the results disagree with the naive estimates based on the chiral loop approach. The calculation of $d_n(\theta)$ using QCD sum rules will certainly help to resolve this controversy.

The approach we shall use follows recent work [14] on the θ -induced ρ -meson EDM in reducing the operator product expansion to a set of vacuum condensates taken in an electromagnetic and topologically nontrivial background. Expansion to first order in θ results in the appearance of matrix elements which can be calculated via the use of current algebra [15, 16]. In this approach the θ -dependence arises with the correct quark mass dependence, and the relation to the U(1) problem becomes explicit as $d_n(\theta)$ vanishes when the mass of the U(1) “Goldstone boson” is set equal to the mass of pion.

The starting point for the calculation is the correlator of currents $\eta_n(x)$ with quantum numbers of the neutron in a background with nonzero θ and an electromagnetic field $F_{\mu\nu}$,

$$\Pi(Q^2) = i \int d^4x e^{iq \cdot x} \langle 0 | T \{ \eta_n(x) \bar{\eta}_n(0) \} | 0 \rangle_{\theta, F}, \quad (1)$$

where $Q^2 = -q^2$, with q the current momentum.

Firstly, it is important to note that if CP-symmetry is broken by a generic quark-gluon CP-violating source (θ -term in our case), the coupling between the physical state (neutron) described by a spinor v and the current η_n acquires an additional phase factor

$$\langle 0 | \eta_n | N \rangle = \lambda U_\alpha v, \quad U_\alpha = e^{i\alpha\gamma_5/2}. \quad (2)$$

The existence of this unphysical phase α is apparent already when one considers the sum rule for the neutron

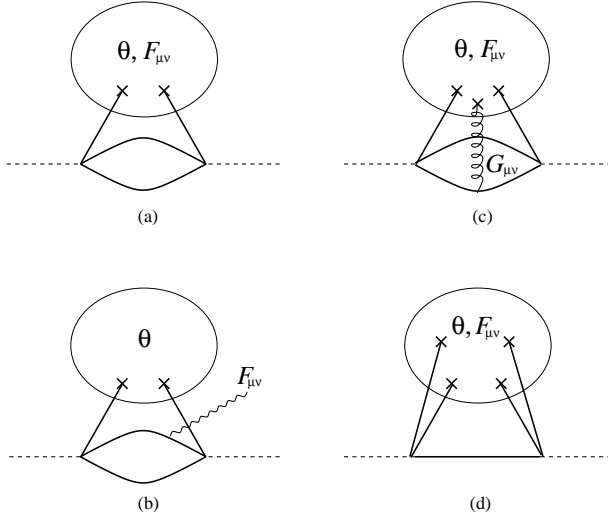


FIG. 1: Various contributions to the CP-odd structure $\{\tilde{F}\sigma, \not{q}\}$. (a) is the leading order contribution while (b), (c) and (d) contribute at subleading order.

mass, which in the absence of CP-invariance can have an additional Dirac structure proportional to $i\gamma_5$. When we turn to electromagnetic form factors, this angle can mix electric (d) and magnetic (μ) dipole moment structures and complicate the extraction of d from the sum rule. Consequently, in general the neutron double pole contribution on the phenomenological side of the sum rule will be proportional to the following expression

$$U_\alpha(\not{q} + m_n)(\mu F\sigma - d\tilde{F}\sigma)(\not{q} + m_n)U_\alpha = m_n\{\mu F\sigma - d\tilde{F}\sigma, \not{q}\} + \sum(\mu \pm \alpha d)\mathcal{O}(F) + \sum(d \pm \alpha\mu)\mathcal{O}(\tilde{F}), \quad (3)$$

in which $\mathcal{O}(F)$ and $\mathcal{O}(\tilde{F})$ are operators depending on the electromagnetic field strength F and its dual \tilde{F} which contain an even number of γ -matrices. We see that only Lorentz structures with an odd number of γ -matrices are independent of α . In calculating d_n , it is then clear that we should study the operator $\{\tilde{F}\sigma, \not{q}\}$, as this is the unique choice with a coefficient independent of α .

The interpolating current η_n is conveniently parametrised in the form,

$$\eta_n = j_1 + \beta j_2, \quad (4)$$

where the two contributions are given by

$$j_1 = 2\epsilon_{abc}(d_a^T C \gamma_5 u_b) d_c \quad (5)$$

$$j_2 = 2\epsilon_{abc}(d_a^T C u_b) \gamma_5 d_c. \quad (6)$$

j_2 vanishes in the nonrelativistic limit, and lattice simulations have shown that j_1 indeed provides the dominant projection onto the neutron (see e.g. [17]). From (2), we may define the coupling to the neutron state in the form $\langle 0|\eta_n|N\rangle = (\lambda_1 + \beta\lambda_2)v$.

Within the sum rules formalism, one has the imperative of suppressing the contribution of excited states and

higher dimensional operators in the operator product expansion (OPE), and thus its convenient to choose β to this end. Ioffe has shown that $\beta = -1$ is an apparently optimal choice for the mass sum rule. However, it is clear that this optimization may differ for different physical observables. We shall therefore keep β arbitrary, and optimize once we have knowledge of the structure of the sum rule.

We now proceed to study the OPE associated with (1). The relevant diagrams we need to consider are shown in Fig. 1 ((a), (b) and (c)). Diagrams of the form (d), although suffering no loop factor suppression, are nonetheless suppressed due to combinatorial factors and the small numerical size of $(\langle \bar{q}q \rangle)^2$.

In parametrizing θ , we shall take a general initial condition in which a chiral rotation has been used to generate a γ_5 -mass, so that

$$\mathcal{L} \sim \dots - \theta_q m_* \sum_f \bar{q}_f i\gamma_5 q_f + \theta_G \frac{\alpha_s}{8\pi} G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a + \dots, \quad (7)$$

in which we restrict to $q_f = u, d$, and so $m_* = m_u m_d / (m_u + m_d)$. The physical parameter is of course $\bar{\theta} = \theta_q + \theta_G$, but we shall keep the general form (7) and calculate the OPE as a function of *both* phases. The independence of the final answer of $\theta_q - \theta_G$ will provide a nontrivial check on the consistency of our approach. We shall find that this requires the consideration of mixing with additional currents, a point we shall come to shortly.

The vacuum structure is conveniently encoded in a generalised propagator expanded in the background field and the associated condensates. We work as usual with a constant background electromagnetic field, so that $A_\mu(x) = -\frac{1}{2}F_{\mu\nu}(0)x^\nu$, and use a fixed point gauge for the gluon potential, $A_\mu^a(x) = -\frac{1}{2}G_{\mu\nu}^a(0)t^a x^\nu$. The electromagnetic field dependence is determined in terms of the magnetic susceptibilities χ , κ and ξ , defined as [10]:

$$\begin{aligned} \langle 0|\bar{q}\sigma_{\mu\nu}q|0\rangle_F &= \chi_q F_{\mu\nu} \langle 0|\bar{q}q|0\rangle \\ g\langle 0|\bar{q}(G_{\lambda\sigma}^a t^a)q|0\rangle_F &= \kappa_q F_{\mu\nu} \langle 0|\bar{q}q|0\rangle \\ 2g\langle 0|\bar{q}\gamma_5(\tilde{G}_{\lambda\sigma}^a t^a)q|0\rangle_F &= i\xi_q F_{\mu\nu} \langle 0|\bar{q}q|0\rangle, \end{aligned}$$

while the θ -dependence is either explicit in the case of θ_q , or extracted via use of the anomalous Ward identity (see e.g. [16]) in the case of θ_G . The θ -dependence of the OPE then follows as a consequence of $m_\eta \gg m_\pi$, and disappears when $U(1)$ -symmetry is restored ($m_\eta \rightarrow m_\pi$) [14].

Defining $iS(q) \equiv \langle 0|q_a(x)\bar{q}_b(0)|0\rangle_{F,\theta}$, and ignoring a trivial δ -function over colour indices, the leading order propagator adapted to the CP-odd sector and appropriate for Fig. 1(a), takes the form,

$$\begin{aligned} S_{LO}(x) &= \frac{\not{x}_{ab}}{2\pi^2 x^4} - \frac{m_*}{4\pi^2 x^2} (1 - i\theta_q \gamma_5)_{ab} \\ &\quad - \frac{\tilde{\chi}_q m_* \bar{\theta}}{24} F_{\alpha\beta} x_\alpha (\gamma_\beta \gamma_5)_{ba} - \frac{\tilde{\chi}_q}{24} (F\sigma(1 + i\theta_G \gamma_5))_{ba}, \quad (8) \end{aligned}$$

where we have denoted $\tilde{\chi}_q = \chi_q \langle \bar{q}q \rangle$, and we shall henceforth follow [10] and assume that $\chi_q = \chi e_q$ etc., with flavour independent parameters χ, κ, ξ .

Diagrams (b) and (c) in Fig. 1 require, in addition, the leading order expansion in the background gluon and electromagnetic fields, and S_{NLO} is consequently more involved. However, since the approach is analogous to that for the leading order terms, we shall defer full details [18], and simply present the final result for the OPE structure arising from these diagrams. In momentum space we find

$$\begin{aligned} \Pi(Q^2) = & -\frac{\bar{\theta}m_*}{64\pi^2} \langle \bar{q}q \rangle \{ \tilde{F}\sigma, \not{q} \} \left[\chi(\beta+1)^2(4e_d - e_u) \ln \frac{\Lambda^2}{Q^2} \right. \\ & - 4(\beta-1)^2 e_d \left(1 + \frac{1}{4}(2\kappa + \xi) \right) \left(\ln \frac{Q^2}{\mu_{IR}^2} - 1 \right) \frac{1}{Q^2} \\ & \left. - \frac{\xi}{2} ((4\beta^2 - 4\beta + 2)e_d + (3\beta^2 + 2\beta + 1)e_u) \frac{1}{Q^2} \dots \right], \quad (9) \end{aligned}$$

where the first, second, and third lines correspond to contributions from diagrams (a), (b)+(c), and (c), in Fig. 1, and μ_{IR} is an infrared cutoff. The diagrams (d) which contribute at subleading order in q^2 are more problematic because they involve correlators not calculable in the chiral approach, e.g.

$$\Pi_{(d)} \sim \int d^4x e^{iq \cdot x} \langle 0 | T \{ (\bar{q}q)^2(x), m_* \sum_f \bar{q}_f i\gamma_5 q_f \} | 0 \rangle. \quad (10)$$

We estimate such contributions via saturation with the physical η meson. The result turns out to be parametrically smaller than any term listed in Eq. (9).

At different stages of the calculation we observe the appearance of the unphysical phase $\theta_G - \theta_q$ which is closely related with the non-invariance of the currents j_1 and j_2 under a chiral transformation. This invariance is, in fact, restored when we consider the mixing of j_1 and j_2 with another set of currents, $i_1 = 2\epsilon_{abc}(d_a^T C u_b)d_c$ and $i_2 = 2\epsilon_{abc}(d_a^T \gamma_5 C u_b)\gamma_5 d_c$. The mixing between these two sets is explicitly proportional to $\theta_G - \theta_q$. When properly diagonalized on the sum rule for \not{q} , the linear combination of these two sets of currents provides additional contributions proportional to $\theta_G - \theta_q$, exactly cancelling the unphysical piece of $\Pi(Q^2)$. Consequently, it is equivalent to take $\theta_q = \theta_G$ as the most convenient basis when working with j_1 and j_2 where the unwanted phase, and the mixing between $j_{1,2}$ and $i_{1,2}$, are simply absent. This situation resembles the problem in obtaining the “correct” quark mass behavior for the EDM of ρ , addressed in detail in [14].

Inspection of the expression (9) shows that the standard choice of $\beta = -1$ appropriate for CP-even sum rules will not be useful here as it removes the leading order contribution. In general there are two motivated techniques for fixing the mixing parameter β : (1) at a local extremum; or (2) to minimize the effects of the continuum and higher dimensional operators. We find in this

case that extremizing in the parameter β also leads to the unappealing cancelation of the leading order contribution. Thus the most natural procedure appears to be to choose β in order to cancel the subleading infrared logarithm which is ambiguous as a result of the required infrared cutoff. This procedure actually mimics the original motivation for $\beta = -1$ in the CP-even case. We then take $\beta = 1$, and it is this choice that we shall now contrast with the phenomenological side of the sum rule. It is worth to remarking, however, that use of the “lattice” current with $\beta = 0$ will also produce a numerically similar result.

On the phenomenological side of the sum rule we have

$$\Pi^{phen} = \frac{1}{2} \{ \tilde{F}\sigma, \not{q} \} \left(\frac{\lambda^2 d_n m_n}{(q^2 - m_n^2)^2} + \frac{A}{q^2 - m_n^2} \dots \right). \quad (11)$$

We retain here double and single pole contributions, the latter corresponding to transitions between the neutron and excited states, but the exponentially suppressed continuum contribution will be ignored as it is not, in this case, sign definite. Here we use $\lambda = \lambda_1 + \beta\lambda_2$ and A is an effective constant parametrizing the single pole contributions.

After a Borel transform of (9) and (11) and using $\beta = 1$ as discussed earlier, we obtain the sum rule

$$\begin{aligned} \lambda^2 m_n d_n + A M^2 = & -\frac{M^4}{32\pi^2} \bar{\theta} m_* \langle \bar{q}q \rangle e^{m_n^2/M^2} \\ & \times \left[4\chi(4e_d - e_u) - \frac{1}{2M^2} \xi(4e_d + 8e_u) \right]. \quad (12) \end{aligned}$$

The coupling λ present in (12) may be obtained from the well known sum rules for the tensor structures **1** and \not{q} in the CP even sector (see e.g. [17] for a recent review). We shall construct two sum rules in this way.

- **(a)** Firstly, we extract a numerical value for λ via a direct analysis of the CP even sum rules. This analysis has been discussed before and will not be reproduced here (see e.g. [17]). One uses $\beta = -1$, and obtains $(2\pi)^4 \lambda \sim 1.05 \pm 0.1$.
- **(b)** As an alternative, we extract λ explicitly as a function of β from the CP-even sum rule for \not{q} , and substitute the result into (12) choosing $\beta = 1$.

The conventional approach which we shall adopt here is to assume that A is independent of M , and thus the left hand side of (12) is linear in M^2 provided that λ is constant in the appropriate region for the Borel parameter. The latter point in case (b) above may be checked explicitly.

It is convenient to define an additional function $\nu(M^2)$,

$$\nu(M^2) \equiv \frac{1}{2\bar{\theta}m_*} \left(d_n + \frac{A M^2}{\lambda^2 m_n} \right), \quad (13)$$

which is then determined by the right hand side of (12).

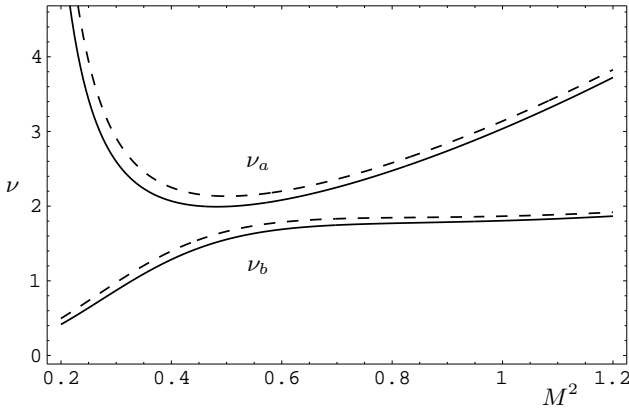


FIG. 2: The neutron EDM function $\nu(M^2(\text{GeV}^2))$ is plotted according to the sum rules (a) and (b). The dashed line shows the contribution from the leading order term only.

The two sum rules described above for ν_a and ν_b are plotted in Fig. 2, where the effect of the higher dimensional terms in (12) proportional to ξ is also displayed. $\nu(M^2)$ is to be interpreted as a tangent to the curves in Fig. 2. For numerical calculation we make use of the following parameter values: For the quark condensate, we take $\langle 0|\bar{q}q|0\rangle = -(0.225 \text{ GeV})^3$, while for the condensate susceptibilities, we have the values $\chi = -5.7 \pm 0.6 \text{ GeV}^{-2}$ [19], and $\xi = -0.74 \pm 0.2$ [13]. Note that χ , which enters at $O(1/M^2)$, since it is dimensionful, is numerically significantly larger than ξ . In extracting λ in case (b) we also set a relatively large continuum threshold $s_0 = (2\text{GeV})^2$ for consistency with the CP-odd sum rule in which this continuum is ignored.

One observes that both sum rules have extrema consistent to $\sim 50\%$, suggesting that our procedure for fixing the parameter β is appropriate. Furthermore, the differing behaviour away from the extrema implies that for consistency we must assume A to be small. One then finds $d_n \sim 2\bar{\theta}m_*\nu(M^2 \sim 0.5\text{GeV}^2)$. It is also interesting that the effective scale is around $M \sim 0.7\text{GeV}$ which is well below m_n , and should be cause for concern regarding the convergence of the OPE. Nonetheless, one sees that the corrections associated with the leading higher dimensional operators are still quite small. This low scale is also the reason we have ignored leading-log estimates for the anomalous dimensions, as their status is unclear at this scale. A naive application leads to a small correction that we shall subsume into our error estimate.

Extracting a numerical estimate for d_n from Fig. 2, and determining an approximate error, we find the result[20]

$$d_n = \frac{(1.0 \pm 0.3)}{(500\text{MeV})^2} \bar{\theta} e m_*, \quad (14)$$

for the neutron EDM, for which the dominant contribution naturally arises from χ .

Comparison with the result of ref. [6] indicates rather good agreement in magnitude, due essentially to the low

effective mass scale $M \sim 700\text{MeV}$. The relation between the calculation presented in this letter and the chiral logarithm estimate will be discussed in more detail elsewhere [18], but we note in passing that the sum rule result is parametrically enhanced ($O(1/N)$ vs. $O(1/N^2)$) at large N_c .

Combining our result with the recently improved experimental bound on d_n [4] we derive, allowing a generous 50% uncertainty, the limit on theta:

$$|\bar{\theta}| < 3 \times 10^{-10}, \quad (15)$$

which is quite close to previous bounds.

In conclusion, we have presented a QCD sum rules calculation of the θ -induced neutron EDM. This result is explicitly tied to a set of vacuum correlators which are non-vanishing only in the absence of a U(1) “Goldstone boson”. The use of QCD sum rules in the chirally invariant channel allowed us to unambiguously extract $d_n(\theta)$, and independence of the answer from any particular representation of the theta term (7) was checked explicitly.

Acknowledgments We would like to thank M. Shifman and A. Vainshtein for valuable discussions and comments.

-
- [1] I.B. Khriplovich and S.K. Lamoreaux, “*CP Violation Without Strangeness*”, Springer, 1997.
 - [2] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. **B147** 385; 448 (1979).
 - [3] K.F. Smith *et al.*, Phys. Lett. **B234** 191 (1990); I.S. Altarev *et al.*, Phys. Lett. **B276** 242 (1992).
 - [4] P.G. Harris *et al.*, Phys. Rev. Lett. **82** 904 (1999).
 - [5] V. Baluni, Phys. Rev. **D19** 2227 (1979).
 - [6] R.J. Crewther *et al.* Phys. Lett. **B88** 123 (1979); **91** 487(E) (1980).
 - [7] S. Aoki and T. Hatsuda, Phys. Rev. **D45** 2427 (1992).
 - [8] see e.g. R.N. Mohapatra, Phys. Atom. Nucl. **61**, 963 (1998) and references therein.
 - [9] I. Bigi and N.G. Uraltsev, Sov. Phys. JETP **100** 198 (1991); M. Pospelov, Phys. Rev. **D58** 097703 (1998).
 - [10] B.L. Ioffe and A.V. Smilga, Nucl. Phys. **B232** 109 (1984).
 - [11] I.I. Balitsky and A.V. Yung, Phys. Lett. **B129** 328 (1983).
 - [12] V.M. Khatsimovsky, I.B. Khriplovich, and A.S. Yelkhovsky, Ann. Phys. **186** 1 (1988).
 - [13] I.I. Kogan and D. Wyler, Phys. Lett. **B274** 100 (1992).
 - [14] M. Pospelov and A. Ritz, hep-ph/9903553.
 - [15] R.J. Crewther, Phys. Lett. **70B** 349 (1977).
 - [16] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. **B166** 493 (1980).
 - [17] D. B. Leinweber, Ann. Phys. **254**, 328 (1997).
 - [18] M. Pospelov and A. Ritz, hep-ph/9908508.
 - [19] V.M. Belyaev and Ya.I. Kogan, Sov. J. Nucl. Phys. **40** 659 (1984).
 - [20] (04/2005) v3: This updated expression corrects an overall factor of two error present in previous versions.